

# Evaluation of a Three-Phase Distribution System State Estimation for Operational Use in a Real Medium Voltage Grid

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**Abstract**— The aim of the presented work is to develop an approach for the proper assessment of pseudo measurements for a three-phase distribution system state estimation under the conditions of a real medium voltage grid. The evaluation is based on the validation of the voltage estimation error, which describes the difference between the calculated node voltages determined by the distribution system state estimation and the actually measured node voltages, as a measure of the quality of the calculated results. By replacing real measurement values with pseudo measurements, the influence on the quality of the three-phase distribution system state estimation for incomplete measurement acquisition will be investigated. For this purpose, customizable synthetic profiles are used to replace real power values in this work.

**Index Terms**— three-phase distribution system state estimation, real measurement data, pseudo measurements, voltage estimation error, meter placement, medium voltage.

## I. INTRODUCTION

The increase in decentralized renewable energy sources (RES) has in recent years progressively led to the fact that electricity no longer exclusively flows from the voltage levels of the transmission system to the subordinate voltage levels of the distribution system, in order to supply the end customer, as it used to be in the classic electrical energy supply. Furthermore, depending on the local feed-in situation, a load flow reversal on the distribution grid side from the lower grid levels to the upper grid levels increases. A particularly high level of these multidirectional load flows can be recognized by the constant expansion of photovoltaic systems (PVS) in the low-voltage grids as well as wind turbines (WT) in medium and high voltage grids. Other aspects that need to be considered

with regard to changing load flow situations result from the growing number of electric cars as well as the increasing use of storage facilities, e.g. for self-consumption optimization. On the one hand, this leads to new simultaneities, which can result in new peak loads, which in turn lead to previously unknown utility loading. On the other hand, distribution system operators (DSOs) are encouraged to increase the efficiency of integrating RES and new types of consumers through the use of smart technologies and structures, such as feed-in and load-management. The progressively changing feed-in and load-situations lead permanently to sustainably changed supply tasks for the DSOs, which will in the short or long term require a much more detailed knowledge of the current system state in the distribution grids.

## II. CHALLENGES

In the transmission system, the system state has been reliably determined by state estimations (SE) since the 1970s [1]. However, the different framework conditions, such as the network topology, network impedance or missing measurement values, prevented an immediate adoption of the concept of SE in the distribution grids. The adaptation of the transmission system SE to the distribution system has been the subject of research for quite some time. Thereby, it has been shown that in particular the small number of measurement points in the distribution grids, especially at the medium and low voltage level, is occasionally one of the biggest hurdles for the area-wide use of SE in these voltage levels. The monitoring of a power system is usually associated with a high financial investment. In particular, investment costs in measuring systems and in information and communication technologies play a decisive role. For this reason, it raises questions on the DSO side, regarding the required number of measurement points in a monitored grid area, to establish complete observability at all grid nodes, even if the considered grid is not completely measured. This in turn raises the question of the quality of a SE with a reduced number of real input data. For this reason, the research project grid-control [2] offers the possibility to implement a Distribution System State Estimation

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(DSSE) for a real medium and low voltage grid and to validate the quality of the DSSE under real framework conditions by means of measured data.

### III. STATE OF THE ART

#### A. Methodes for State Estimation in Distribution Grids

Several methods for DSSE have been investigated in the last years, with the most widely optimization concepts Weighted Least Squares Estimator (WLS), Weighted Least Absolute Value Estimator (WLAV) and Schweppe-Huber Generalized-M Estimator (SHGM). The most common technique, the WLS method, has been already widely investigated in [7], [8], [9] and [10]. The performance of WLS was compared with the other algorithms based on different criteria. They ascertained that the WLS algorithms give best performance under Gaussian assumptions for known noise characteristics and, therefore, it is the most suitable method for the use in distribution systems. Due to the mentioned research results, the algorithm presented in this paper relies on a WLS method adjusted for power systems as presented for example by Abur and Exposito [3]. The state estimation algorithm serves to solve the minimization problem in equation (1), which describes the weighted sum of squares between the calculated and the measured values.

$$\text{Min } J(\mathbf{x}) = [\mathbf{z} - \mathbf{h}(\mathbf{x})]^T \cdot \mathbf{R}^{-1} \cdot [\mathbf{z} - \mathbf{h}(\mathbf{x})] \quad (1)$$

The above equation describes the objective function  $J(\mathbf{x})$  of the DSSE, where  $\mathbf{z}$  describes the measurement vector and  $\mathbf{h}(\mathbf{x})$  the vector of the nonlinear functions relating the measurements to the state variables in vector  $\mathbf{x}$ . In addition,  $\mathbf{R}$  describes the weighting matrix of the measurement values as a function of the measurement value accuracy. The goal of the DSSE is to determine the state variables  $\mathbf{x}$  based on the available measurement values. The determination of the state variables (complex node voltages) takes place by deriving the objective function  $J(\mathbf{x})$ , which can be described as follows:

$$\left[ \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} \right] = -2\mathbf{H}^T(\mathbf{x}) \cdot \mathbf{R}^{-1} \cdot [\mathbf{z} - \mathbf{h}(\mathbf{x})] \quad (2)$$

In equation (2),  $\mathbf{H}$  describes the Jacobian matrix in which the mathematical functions of the used measurements derived from the state variables are represented. The condition for minimizing equation (2), is given below in equation (3).

$$\mathbf{H}^T(\mathbf{x}) \cdot \mathbf{R}^{-1} \cdot [\mathbf{z} - \mathbf{h}(\mathbf{x})] = 0 \quad (3)$$

In order to determine the state variables in  $\mathbf{x}$ , equation (3) must be solved for  $\mathbf{x}$ . This is initially not possible due to the nonlinearity of the measurement data model defined in  $\mathbf{h}(\mathbf{x})$ . In order to be able to change the equation nevertheless,  $\mathbf{h}(\mathbf{x})$  is developed for a linearization in a Taylor series. Since  $\mathbf{H}$  already describes the first derivation of the calculated measurements with respect to the state variables, the Taylor series can be represented as follows.

$$\mathbf{h}(\mathbf{x}) = \mathbf{h}(\mathbf{x}_0) + \mathbf{H}(\mathbf{x}_0)\Delta\mathbf{x} \quad (4)$$

By substituting equation (4) into equation (2), equation (5) is obtained. It describes the target function of the DSSE, which was linearized by a Taylor series and developed around the start value  $\mathbf{x}_0$ .

$$[\mathbf{H}^T(\mathbf{x}_0)\mathbf{R}^{-1}\mathbf{H}(\mathbf{x}_0)]^{-1}\mathbf{H}^T(\mathbf{x}_0)\mathbf{R}^{-1}[\mathbf{z} - \mathbf{h}(\mathbf{x}_0)] = \Delta\mathbf{x} \quad (5)$$

Using an iterative calculation of equation (5) after Newton-Raphson, the changes in the state variables are determined until they fall below a given threshold [3], [4], [5]. Equation (5) describes in matrix notation an equivalent formulation to the Maximum-Likelihood-Estimator for normal distributed measurement errors, on which the WLS method is based [6]. This procedure is used to determine the most likely system state considering the measurement errors.

#### B. Algorithm for Three-Phase State Estimation in Distribution Grids

The algorithm selected for the three-phase DSSE in this work is based on the formulation of the complex node voltages in rectangular formulation as proposed in [11]. The advantage of this method is the use of all types of measurement data. However, unlike the use of polar node voltages, the equations are based on currents instead of power. For this procedure, the power measurements are therefore converted into equivalent current measurements in each iteration. The partial derivatives of the measurement model result in a matrix with exclusively constant entries which correspond to the real and imaginary parts of the node admittance matrix, i. e. the values for the conductivity  $G$  and the susceptance  $B$ , and remain the same in each iteration step.

### IV. GRIDLABORATORY WITHIN THE RESEARCH PROJECT GRID-CONTROL

Figure 1 shows the grid laboratory of the research project grid-control. The grid laboratory consists of one medium voltage feeder, which supplies 43 local grids. 34 substations of the local grids are measured at the low-voltage busbar of the transformer and recorded with a time resolution of one minute. The recorded measurement types are active power, reactive power, phase-to-phase voltage, conductor-to-ground voltage and current.

#### A. Composition of the Gridlaboratory

Figure 2 shows in the upper part, a schematic representation of the medium voltage feeder from Figure 1. It illustrates the switching substation and an exemplary composition of the supplied local grids. In this case the switching substation is part

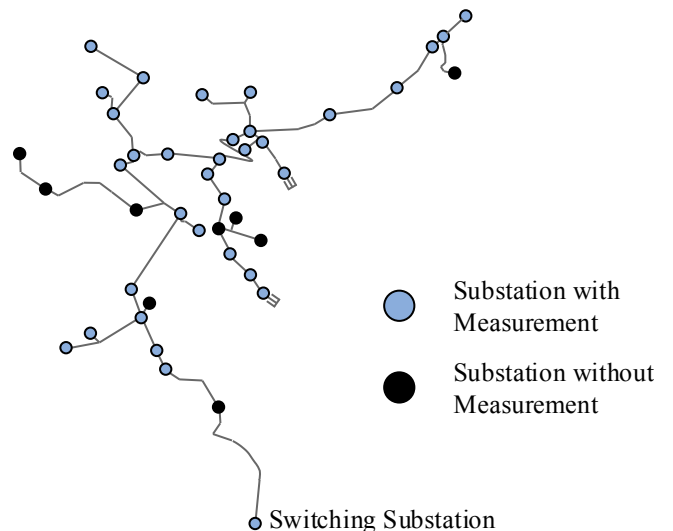


Figure 1. Topology of the grid in grid-control

of an upstream 20 kV medium voltage grid. Furthermore, the measurement devices are indicated at the low voltage busbars of the transformers connected to the local grids. The local grids supplied by the 20 kV feeder are of varying size in terms of the connected number of loads, load types, numbers of RES generators and installed RES power. The occurring load types are households, agricultures and commercials, the occurring RES generator types are PVS and WT. In general, 927 household, 78 agriculture and 81 commercial loads are supplied by the 20 kV feeder. This is countered by 197 PVS with an installed power of approx. 2.7 MW and one WT with an installed power of 1.8 MW.

## V. MODEL PARAMETERIZATION FOR THE EVALUATION OF THE DSSE

### A. Aggregated Synthetic Load and Infeed Values for the local grids

Due to missing measurement points in medium and low voltage distribution grids it has become common in the process of DSSE to use substitute values, so called pseudo measurements, in compensation. The lower part of Figure 2 illustrates the same medium voltage feeder as in the upper part. In contrary to the upper part, the supplied local grids are simulated by load and infeed pseudo measurements which are directly connected to the medium voltage feeder. The depicted gaussian distribution (green dashed line) besides the outlined load and infeed mockups in the lower part of Figure 2, indicates the error due to the pseudo measurements. They are comparable with the measurement error of a measuring device but unless they are not really measured but only assumed as measured, they have a much wider curve around the expected value, so that they will be treated as less trustworthy input data compared to real measured input data. As indicated above not all of the 43 local grids are measured. The 9 unmeasured local grids are replicated using pseudo measurements from the beginning. The synthetic load profiles used as pseudo measurements in this work are based on the method proposed in [13] and [14]. In [13] the comparison of stochastic load profile modeling approaches for low voltage residential consumers is carried out, to remodel the characteristic of real smart meter measurement data. In this work the approach based on a Markov chain is used. In [14] the adaption of synthetic load profiles with regard to the maximum power and the annual energy consumption for a better simulation of the consumer behavior is described. This is done by a Power-Energy-Denormalization (PED) method, which aims to calculate load profiles, where the maximum power and the annual energy consumption is a degree of freedom and can be chosen by the user. The synthetic infeed values are estimated on the basis of a reference PVS. Scattered measured values are available as well as the technical data of the reference PVS. The measured values will be normalized to the known installed power, to receive a standardized infeed power. This standardized power can then be used to determine the infeed power of the remaining PVS in the grid area. The aggregation for the local grids is done by averaging the individual load or infeed profiles based on their corresponding number. The result of the aggregation process in combination with the PED method is illustrated in Figure 3. The blue solid line represents the measured active power of the residual load. The orange dashed line is the result of the aggregated load method described above.

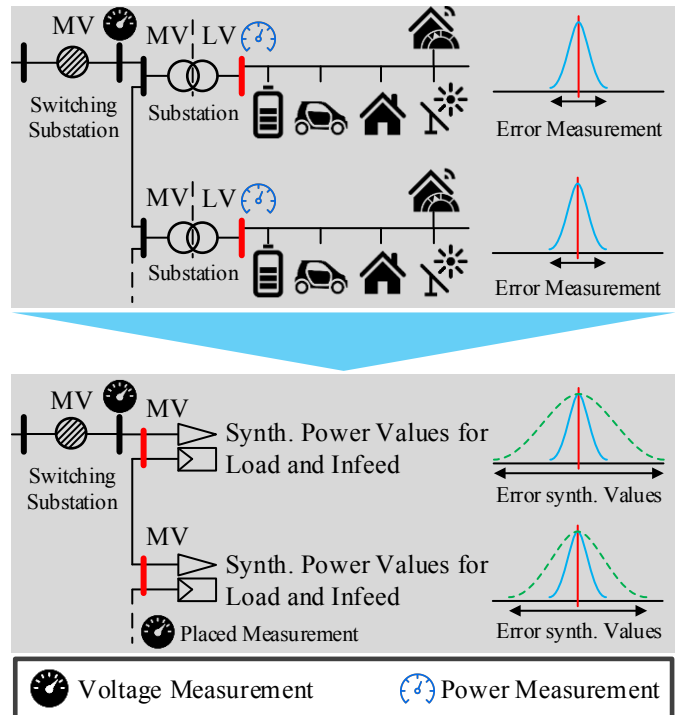


Figure 2. Schematic representation of the medium voltage feeder with real and synthetic values for the supplied local grids

Furthermore, the synthetic infeed active power values of the 9 PVS in the exemplary local grid (green dot dashed line) is shown within the Figure 3. The resulting residual synthetic load curve is represented by the red dotted line. At first glance, the synthetic curve closely approximates the measurements, whereby the characteristic day night distinction is well reproduced. Nevertheless, deviations are clearly visible, especially in the area of the occurring peaks. The fed back provoked by the PVS occurs in the synthetic curve at the same time steps as in the measurements. With regard to the method of weighted least squares, the normal distribution of the errors is an important prerequisite for using the synthetic input data in the VNZS. For this reason, a visual check was made for the normal distribution of the deviations using histograms. Considering, that the synthetic residual load curves do not contain any measurement information, they describe the behavior of the local grid within an acceptable range. In addition, the described procedure thereby allows preserving the previously mentioned normal distribution of the measurement errors and thus allows the application of the synthetic load values in the DSSE without restrictions.

### B. Conversion of the measurement and pseudo measurement accuracy to weighting factor

The depicted gaussian distribution (blue solid line) besides the outlined local grids in the upper part of Figure 2 indicates the accuracy of the measuring devices installed at the low voltage busbars at the substations of the measured local grids. The measurement accuracy provides the information about the quality of the measuring device and thus a measure of deviation from the expected value  $\mu$ . The red vertical line in Figure 2 describes  $\mu$  and the blue marked normal distribution describes the standard deviation  $\sigma$  around the  $\mu$ . In the DSSE algorithm

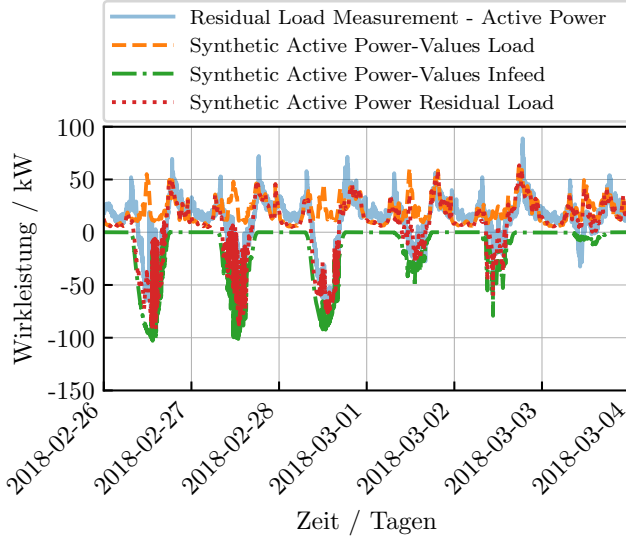


Figure 3. Comparison of real and synthetic residual load curves based on an exemplary local grid of the 20 kV feeder

the information about the measurement error is used to categorize the given measurement input data for the evaluation of how trustworthy the single measurements are. The values thus represent the weighting factors of the weighting matrix  $\mathbf{R}$  in equation (1). Assuming that the range between  $(\mu - 3\sigma)$  and  $(\mu + 3\sigma)$  covers 99.73% area under the Gaussian distribution, equation (6) can be used to convert the accuracy of the measurement to the standard deviation [12].

$$\sigma = \frac{\mu * accuracy}{3 * 100} \quad (6)$$

The accuracy is described by a given percentage of the maximum error around  $\mu$ . The measurement error is set by the meters and is 2% for voltage and 3% for power measurements in this work. The error of the synthetic data compared to the measurement data is determined by the symmetric mean absolute percentage error and is for the residual load curves of the individual local grids in the range up to 150%.

### C. Simulation Scenarios

Due to the minute time resolution of the measured data, a complete annual simulation is therefore not possible in relation to the computation time, since this involves 525,600 time steps to be simulated. For this reason, simulations in the range of one week are first carried out in the period of the highest load whereby the number of time steps to be simulated being reduced to 10,080, thus resulting in an acceptable computation time. The first simulation scenario during the validation process is to determine the baseline condition for the considered medium voltage feeder during the mentioned period. The baseline condition is defined as the system state, which results from using the real recorded measurements as input for the DSSE. The goal behind the baseline is, therefore, to determine the assumed system state based on the real measured values and to define this as a reference for the considered medium voltage feeder. The second simulation scenario during the validation process is therefore to investigate the influence by the use of load and infeed pseudo measurements on the DSSE by comparing the results with the reference simulation. Because of the synthesizing process it can be expected, that there will be

differences between the reference simulation and the pseudo measurement simulation. To approach the initial reference simulation, selectively placed measured values are included into the DSSE in order to reduce the deviations and to achieve the specified quality of the meter placement. This is the third simulation scenario. Referring to the question of the system operators in the context of the required number of measurement points in the grid and their placement, the simulation scenario two represents the assumed case of a non-measured grid, which is to be completely monitored by an appropriate measurement placement.

## VI. EVALUATION

### A. Measure of Quality

The aim of the methodology presented in this work is to obtain a proper assessment of pseudo measurements for a three-phase DSSE under the conditions of a real medium voltage grid. Usually, the load flow to a prevailing load and infeed situation is determined on the basis of the appliance impedances and the complex node voltages existing at this time. Since the complex node voltages change for each load and infeed situation, they provide a good basis for evaluating the influence of the pseudo measurements. As a measure for the quality, therefore, the voltage estimation error according to equation (7) is defined.

$$E_{rel}^{99\%} = \frac{|V|_{DSSE} - |V|_{measurement}}{|V|_{measurement}} \quad (7)$$

The voltage estimation error describes the relative difference between the calculated node voltages of the DSSE and the recorded node voltage measurements at the substations. In agreement with DSOs, the 99% percentile and a deviation of  $\pm 1\%$  for the calculated node voltage in comparison to the measured voltage value was defined as sufficiently accurate as a guideline.

### B. Results

Figure 4 shows the result for the baseline simulation scenario. Usually the switching substation is used as the slack node during the calculation. In this case, the voltage measurements on the switching substation cannot be used due to erroneous measurements, which is why the next measured substation in the feeder is used as the slack node. At the slack node, the voltage measurements are given as input data for the DSSE. The other measuring points marked in Figure 1 are set with the actually measured active and reactive power values as input data for the DSSE. The maximum errors for the real measurements are chosen as described in section V.B. On the abscissa in Figure 4, the estimation error of the voltage magnitude in p. u. is illustrated and, on the ordinate, the relative frequency of occurrence. The results for the three phases are given by the three different curves, at which each curve consisting of 340,000 data points, which result from the simulation of a week on the basis of minute values. The gray area describes the already mentioned tolerance range of  $\pm 1\%$  deviation around the measured value. The numbers in brackets in the legend reflect the amount of deviations of the 99<sup>th</sup> percentile in the tolerance range. In the basic simulation, more than 99% of all deviations between the calculated and measured voltage magnitude are within the tolerance range. The maximum errors occur in the range of 4.6%, but with a



probability of occurrence of less than 1%. Figure 5 shows the result for the second simulation scenario. As with the baseline simulation, the second measured station will be used as the slack node. In this scenario the only real measurement as input data for the DSSE is the voltage measurement at the slack node. The other measuring points marked in Figure 1 are now set as pseudo measurements with the synthetic residual load curves adapted for each local grid as shown in Figure 3. The maximum errors for the pseudo measurements are chosen as described in section V.B. Compared to the baseline simulation, over 99% of all values in the 99<sup>th</sup> percentile range are also in the tolerance range. In contrast to the baseline simulation, the occurrence within the tolerance band is more limited. This indicates that the errors within the tolerance range are larger, as opposed to using the real measurement data. It can also be seen that a greater deviation in the occurrence of the individual phases occurs in comparison to Figure 4. This is due to the symmetric distribution of the pseudo measurements on the individual phases. The unbalanced loading of the real measured data is not reproduced here. The maximum errors occur in the range of 3.2%, also with a probability of occurrence of less than 1%. These results indicate that a precise voltage measurement at the slack node makes a decisive contribution to the estimation of voltage magnitudes at the nodes in the feeder. The assumed node power values seem to have a small influence to that effect. Additional measurements therefore do not seem to be necessary to estimate the node voltage magnitudes in the considered grid. In contrast, with regard to the current flow through the lines, additional real measuring points in the feeder must be taken into consideration. First results show that the node power values have a major impact on the estimation accuracy of current flows, which is why simulation scenario three needs to be further evaluated.

### I. CONCLUSION

The presented approach for evaluating the proper assessment of pseudo measurements done by a reduction of real measurements in the considered grid area gives the opportunity to determine the influence of pseudo measurements on the system state. The results presented in this work show that the presented synthetic residual load curves are suitable for the estimation of the node voltage magnitudes in combination with a real voltage measurement at the slack node, even if the unbalance effect is not considered.

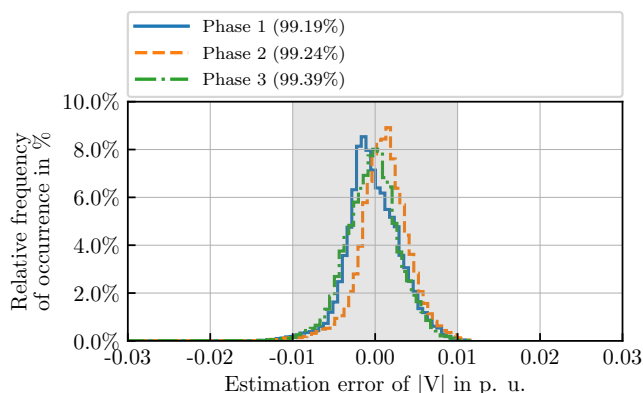


Figure 4. Estimation error of the voltage magnitude for real measured values as input data for the DSSE

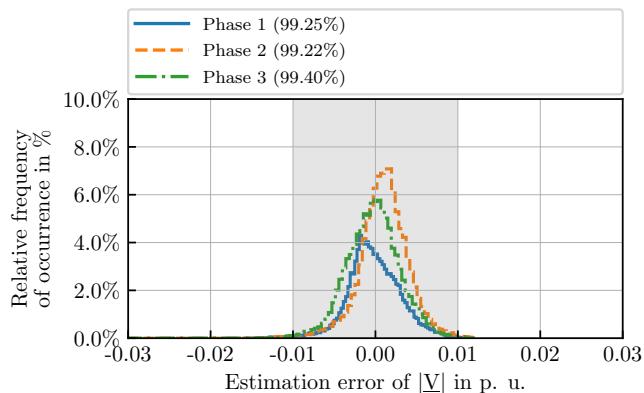


Figure 5. Estimation error of the voltage magnitude for pseudo measurements as input data for the DSSE

First results show, however, that for the estimation of the current flows, the pseudo measurements are not sufficient and therefore further real measuring points in the feeder have to be investigated. Therefore, further investigations regarding the simulation scenario three are necessary. In addition, the method will be tested on another real medium-voltage grid and a real low-voltage grid in which measurement values are recorded in order to verify the results.

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