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ALGORITHMIC DETERMINATION OF POLE-ZERO REPRESENTATIONS OF POWER TRANSFORMERS' TRANSFER FUNCTIONS FOR INTERPRETATION OF FRA DATA

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Abstract: Frequency Response Analysis compares measured transfer functions of power transformers. Deviations of frequency response curves indicate electrical or mechanical damages of windings. As assessments are done by experts, no objective guidelines for interpretation of measurement results exist. This paper deals with approximation of measured power transformer frequency responses using complex rational function models. A fitting algorithm maps the information contained in measured curves on a pole-zero model of reduced complexity. The aim is to develop an algorithm for automated interpretation using analytical models created on the basis of measurement data. Requirements of accuracy of the approximation models for purposes of Frequency Response Analysis are considered. An enhanced algorithm for complex rational function estimation based on the proven Vector Fitting method is presented. Finally, application of the algorithm with real measured data is demonstrated.

1. INTRODUCTION

Frequency Response Analysis (FRA) is one of the key tools in power transformer condition assessment enabling detection of winding and core faults. Its application is mostly meaningful after events such as electrical faults in the connected power grid or after transport of a power transformer. Changes of the transfer function (TF) indicate mechanical and/or electrical changes of the active part. However, interpretation of frequency response deviations is a non-standardised step and is therefore not objective. Mathematic modelling of power transformer transfer functions obtained by measurements is – in this approach – regarded to be the first step for future automatic interpretation of frequency response deviations since assessments can be based on a set of rules evaluating the changes of two analytical transfer function models rather than the measured data.

1.1. Basic principle of FRA

The electrical behaviour of power transformers in higher frequency range is determined through a complex network composed of the resistance and inductance of the windings as well as various capacitances between windings, core and tank. Capacitances play a dominating role in higher frequencies while core effects can only be seen in lower frequency range. Geometrical changes of the windings like displacements, buckling or conductor tilting are changings of distances between energized elements which result in altered capacitances. Moreover, a windings' self-inductivity can vary due to altered flux path structure. So it can be said that a power transformers' frequency response is a unique representation of its structural layout.

FRA is a comparative method: The functional principle is to compare two complex transfer functions with each other. One is the so-called reference TF, another is the test TF, which e.g. has been measured when the transformer was offline during service.
No or slight differences between TFs indicate no electrical or mechanical change inside the transformer while deviations suggest potentially critical variances in relation to the reference. Figure 1 shows an illustration of a power transformer seen as an electrical multi-port network.

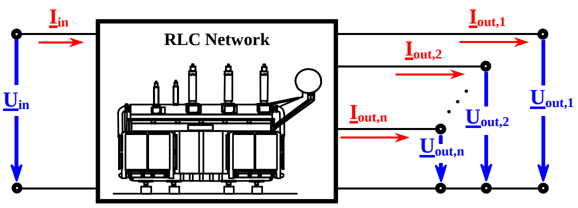

Figure 1: Power transformer as multi-port network.

Particularly three phase transformers offer numerous possible (complex) transfer functions. Regarding one quadripole, two main types are transferred voltage

$$\underline{TF}_{U,out}(f) = \frac{FFT\{u_{out}(t)\}}{FFT\{u_{in}(t)\}} = \frac{\underline{U}_{out}(f)}{\underline{U}_{in}(f)} \qquad (1)$$

and input current (input admittance):

$$\underline{TF}_{I,in}(f) = \frac{FFT\{i_{in}(t)\}}{FFT\{u_{in}(t)\}} = \frac{\underline{I}_{in}(f)}{\underline{U}_{in}(f)} \qquad (2)$$

Concerning one dedicated two-port network, investigations in the past revealed, that all TF types show sensitivity towards mechanical changes [1].

1.2. State of the art and interpretation difficulties

In the past, FRA experts were mainly dedicated to work on reliable measurement techniques that ensure reproducible measurement results. This is an indispensable premise for FRA ensuring its

authenticity as a diagnostic method (see [2]). However, there's a gap in one's knowledge concerning the interpretation of TF deviations.
A visual comparison of the of TF magnitude curves $|\underline{TF}(f)|$ done by experts is state of the art. As yet, only very few rudiments for algorithm assisted interpretation of FRA results exist [3]. Application of these approaches with FRA data measured in the field showed algorithmic shortcomings which confirms that FRA assessments by experts are not replaceable so far.

1.3. FRA interpretation methodologies

Saying interpretation, i.e. (visual) comparison of power transformer frequency responses, special attention to resonance characteristic of TFs has to be paid. Damping differences covering the whole frequency range are typical for variances in the measurement setup. Noticeably frequency confined damping differences, relocation of resonance peaks, creation of new resonances or disappearance of resonances in relation to the reference TF are quite evident for electrical or mechanical changes of the active part of a power transformer. The challenge of future automatic FRA interpretation is to capture these assessment methodologies. Only with algorithmic interpretation, objective evaluation of FRA measurement results is possible. Information about resonance behaviour of TFs lies within a mathematical formulation of frequency responses. The main aim is to compare analytical models of frequency responses in order to grasp deviations of resonances between FRA curves, which is pre-condition for advanced automatic interpretation. Further paragraphs of this paper deal with the analytical formulation of TF by means of an enhanced rational approximation method.

2. APPROXIMATION OF FREQUENCY RESPONSES

2.1. Analytical modelling of TFs

An analytical function is a mathematical formulation using a closed expression. TFs of RLC two-port networks are linear systems and can be described by rational functions consisting of two polynomials with real coefficients (a_m, b_n):

$$TF(s) = \frac{a_0 + a_1 s + \ldots + a_M s^M}{b_0 + b_1 s + \ldots + b_N s^N} \quad (3)$$

where s denotes the complex frequency of a Laplace transformed function in time domain. An equivalent expression to (3) is given by:

$$TF(s) = A \frac{\prod_{m=1}^{M} (s - z_m)}{\prod_{n=1}^{N} (s - p_n)} \quad (4)$$

As (a_m, b_n) are real, zeros and poles (z_m, p_n) come in complex conjugate pairs.

2.2. Rational approximation by Vector Fitting

In [4], an iterative method called "Vector Fitting" (VF) is described which, given the predefined degree of (4), tries to find the best fitting rational function for a measured complex frequency response in a least square sense. The algorithm starts with an arrangement of $N/2$ stable complex conjugate pole pairs equally distributed over the whole measured frequency range and relocates them towards poles of the measured curve within one iteration step to improve the fitting. The algorithm reformulates this nonlinear approximation problem by sequentially solving two linear problems. Figure 2 shows a measured TF magnitude curve together with the magnitudes of the approximation function after 10th iteration step. The red dashed-dotted and blue dotted vertical lines show the corresponding frequency positions of poles after one respectively 10 iteration steps.

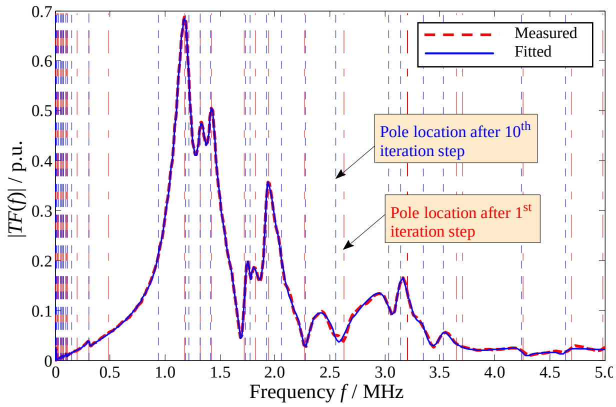

Figure 2: Measured transformer frequency response and approximated functions with corresponding pole locations of different iteration stages.

2.3. Parameters of approximation process

The final result of the approximation process is not independent from user input parameters. The number of iterations and the assumed degree of measured TF play an important role for the algorithm. The degree of the rational function has to be chosen carefully. Although the algorithm converges if the chosen ordinal number N exceeds the needed theoretically needed complexity, the result will contain lots of unimportant poles (with great negative real part) which makes it intransparent. In another word: If a measured frequency response of e.g. 1000 points is fitted by a rational function of degree 600, the result is useless. On the other hand, if the chosen degree is below the needed value, the algorithm will only deliver poor accuracy, which is not acceptable. Both parameters – ordinal number and number of iterations – influence each other. High numbers of iterations do not improve the fitting result necessarily, as shown later in this paper. Additionally, not only ordinal number but also prior choice of starting pole locations has an effect on the final fitting result.

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3. FITTING ACCURACY REQUIREMENTS

In this paper, the aim of frequency response approximation is, to obtain a fitted rational function of minimal complexity that fulfils the requirements in accuracy for FRA curve comparisons. The approach of parameterisation of TF curves for further interpretation purposes will only be successful, if different algorithms produce the same pole-zero representations for the same input data [5]. The need to establish measures along with threshold values assuring approximation quality is evident. Loss of information during fitting process is not acceptable.

3.1. Measuring of approximation accuracy

Quality of approximation can easily be quantified with the measure of root mean square error (RMSE) of standardized TFs:

$$\mathrm{RMSE} = \sqrt{\frac{1}{L}\sum_{\nu=1}^{L}\left(\frac{|\underline{TF}_{orig}(\nu\cdot\Delta f)| - |\underline{TF}_{fit}(\nu\cdot\Delta f)|}{|\overline{TF}_{orig}|}\right)^2} \qquad (5)$$

using

$$|\overline{TF}_{orig}| = \frac{1}{L}\sum_{\nu=1}^{L}|\underline{TF}_{orig}(\nu\cdot\Delta f)| \qquad (6)$$

Standardization cares for comparability of transfer function types with different co-domains. Investigations with many measured transformer frequency responses revealed, that approximations with calculated RMSE in the per mill range deliver sufficient accuracy. The crucial criterion is, that the fitting error is smaller than the deviations caused by measuring tolerances of repeated FRA measurements. The following figure shows two FRA curves of the transferred voltage (phase 1W-2W) of the same transformer.

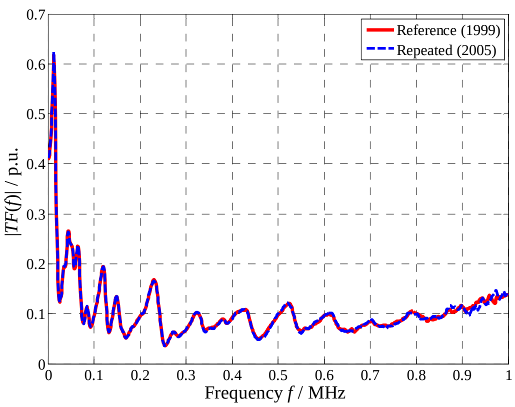

Figure 3: Reference and repeated measurement of transferred voltage on phase W of 200MVA transformer (220kV-110kV) with RMSE = 0.0344.

The time period between reference and repeated measurement is almost 6 years. Calculated RMSE is 0.0344 which indicates very good repeatability; Other measurements exceed the RMSE value of 0.1 but still can be regarded as acceptable.

3.2. Quality of resonance fitting

Not only has the parameterization of measured TF to meet the needs concerning the integrated measure of RMSE, but also the precision of reproducing resonance peaks has to be satisfied. When comparing two frequency response curves, differences of resonance peaks in magnitude, Q and – above all – frequency are important. So as to create rational functions for FRA interpretation to this end, variances of resonances between measured and fitted curve have to be smaller than deviations caused by the measurement respectively resolution of the data recording format. As an example, Figure 4 shows the result of an approximation in the low frequency part of a TF. This fitting result can be regarded as limit case regarding accuracy.

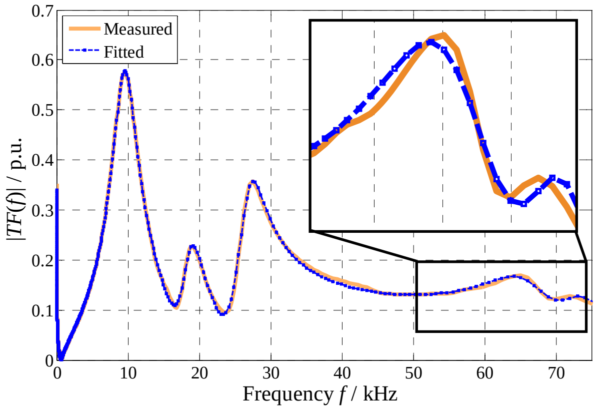

Figure 4: Measured and approximated TF in lower frequency range. Small deviations in frequency of resonances at 65 kHz and 72 kHz can be seen.

A slight shift between resonances at 65 kHz and 72 kHz can be seen. Analysis of many frequency responses – reference and repeated measurements – recorded with equipment of different manufacturers revealed that, depending on measuring parameters like frequency range, number of recorded frequency points, linear or logarithmic distribution of points, shifts of 2...3kHz are within the range of resolution respectively repeatability imprecision. Figure 5 again points out, that fitting quality has to be evaluated by RMSE as well as a comparison of the quality of resonance match.

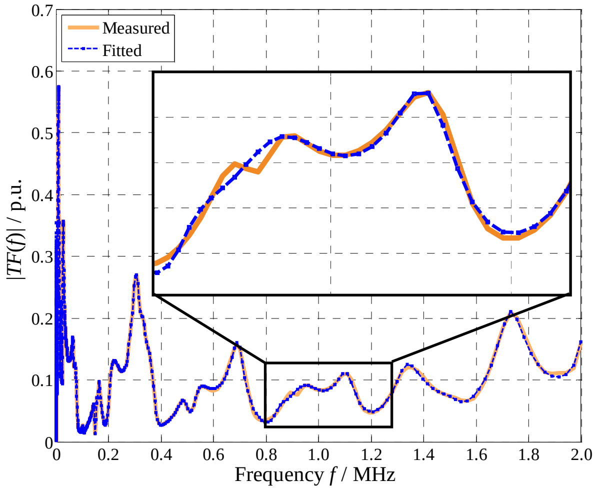

Figure 5: Measured and fitted transfer function with RSME = 0.002

Although RMSE is 0.002 in this case, the two resonances around 920 kHz of the measured curve are not satisfyingly captured by the approximated curve. Automatic recognition of this problem instead of visual check can be challenging. An automatic recognition algorithm for this problem is in early stage of development.

4. ENHANCED FITTING ALGORITHM FOR FRA INTERPRETATION PURPOSES

The Vector Fitting algorithm described in [4] is meanwhile a proven method for creation of mathematic models of measured frequency responses. Applications go from transmission line modeling to extraction of stray parameter functions for dimensioning of filters for electromagnetic compatibility purposes. There are differences between applications concerning frequency range as well as accuracy. As described in previous paragraph, requirements for FRA interpretation considering accuracy are relatively high. This section describes the adaption of the algorithm to improve fitting results.

4.1. Conventional Vector Fitting

Convergence – and therefore fitting quality – of the VF algorithm depends on start conditions. There are two main points to consider: A good (pre-)estimation of the degree of the rational fitting function and a suitable start distribution of poles. Conventional VF requires the desired number $N/2$ of complex conjugate pole pairs as input parameter. The frequency locations of starting poles are by default distributed evenly over the entire frequency range $f_{max} = L \cdot \Delta f$ of the measured TF:

$$p_{n,start} = \alpha_{n,start} + j \cdot \beta_{n,start} \tag{7}$$

with

$$\beta_{n,start} = \frac{\pi f_{max}}{N/2}(2n-1) \tag{8}$$

$$\alpha_{n,start} = -0.01 \cdot \left|\beta_{n,start}\right| \tag{9}$$

($n \in [1, ..., N/2]$ for $f > 0$ denotes one half of N complex conjugate pole pairs).

Figure 6 illustrates the convergence problem encountered with several measured frequency responses. While accuracy in lower frequency range is good, it is insufficient for higher frequencies. Even for high numbers of degree, fitting quality could not be improved. One would think that by increasing the number of iteration steps the fitting result can be improved. Unfortunately this is not an option.

Figure 7 shows the development of RMSE of the frequency response of Figure 6.

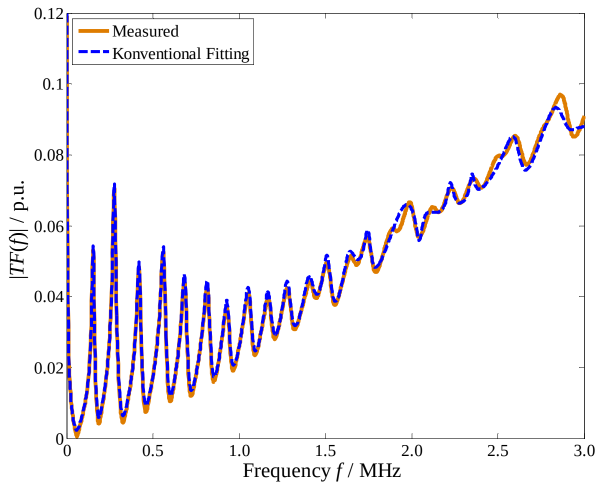

Figure 6: Measured TF and approximation using conventional Vector Fitting. Deviations are obvious.

RMSE stagnates or even increases if pole relocation is repeated – even with very high degree of the fitting rational function. This makes clear that an ill-conditioned starting pole distribution does not lead to satisfying fitting results.

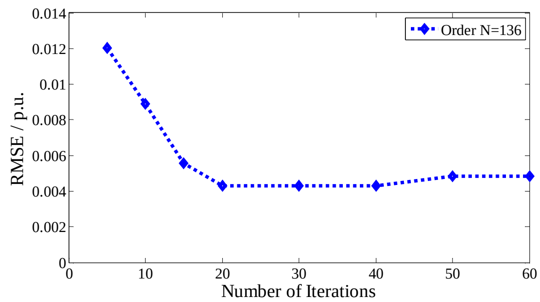

Figure 7: RMSE vs. number of iterations.

This problem can be overcomed by algorithmic estimation of ordinal number and optimization of starting pole distribution.

4.2. Degree estimation of fitting function

The idea behind this approach uses the fact that every pole creates a local resonance peak in $|\underline{TF}(f)|$. Amplitude and Q of local maximum are not only determined by $\alpha_n = \mathrm{Re}\{p_n\}$ but also by adjacent poles that superpose each other. In fact, not every pole creates its corresponding local maximum. However, there exists a certain number of poles between local minima in $|\underline{TF}(f)|$. By segmentation of the frequency range of the measured TF at frequencies with local minima, rough estimation of the needed ordinal number can be achieved. Figure 8 shows the segmentation of a measured TF.

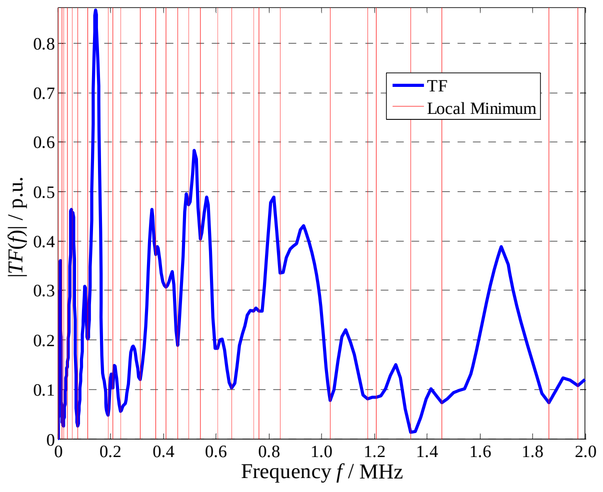

Figure 8: Segmentation of TF using local minima

The precise number of poles between local minima cannot be identified this way but will be determined within the next step. For frequency responses containing mentionable amounts of noise, smoothing of the TF curve using a simple moving average filter helped to find appropriate segmentations.

4.3. Algorithmic determination of starting poles

Because several poles can be located between frequencies of minima, the exact number and location of these poles must be determined for each section. This is done by piecewise executing of the vector fitting algorithm: For every section of $\underline{TF}(f)$ with $f_{min,k} \leq f < f_{min,k+1}$, a fitting rational function is determined, beginning with 2nd order. RMSE is computed after wise. If RMSE < 0.05 is achieved for this section, the found poles are saved in a table. If RMSE > 0.05, the degree of the rational function for this section is increased by 2 and so on, until an appropriate rational function is found for every section. As the final step, Vector Fitting is executed using the determined poles of each section as starting poles. A Nassi-Schneiderman diagram of the algorithm is shown in Figure 9.

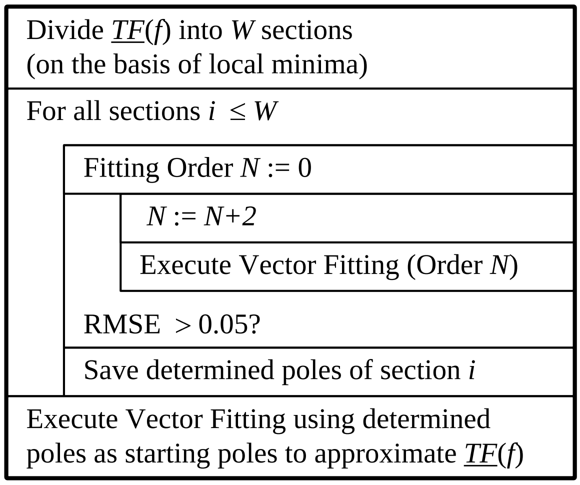

Figure 9: Structogram of enhanced fitting algorithm.

It displays the compactness of the algorithm which can be implemented and reproduced easily.

5. COMPARISON OF FREQUENCY RESPONSES

Figure 10 shows two measured frequency responses recorded logarithmically (800 points). There are obvious deviations beginning at low frequencies.

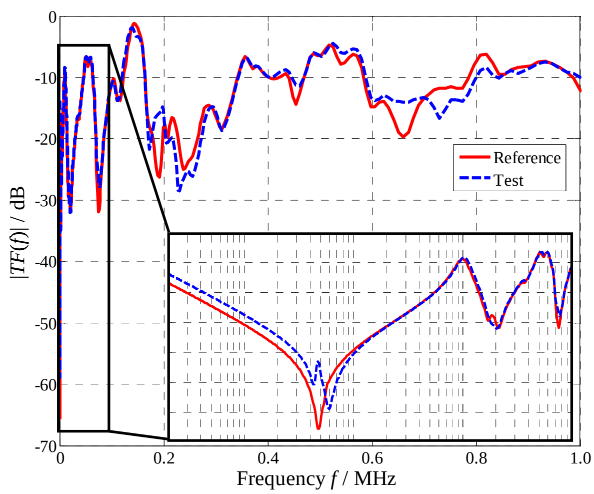

Figure 10: Two deviating frequency responses (800 points, linear-logarithmic view).

A rational function was created for the first frequency response (reference) with accuracy of RMSE < 0.03. After that, a second rational fitting function for the test TF was created using the poles of the first fitting as starting poles. During this process, no new poles are created. The Vector Fitting algorithm relocates these starting poles in order to achieve the most precise approximation to the second TF in a least square sense. Figure 11 shows the fitting result of both frequency responses.

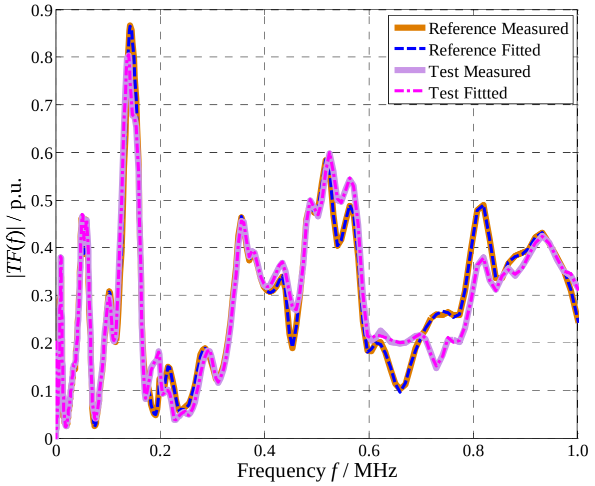

Figure 11: Reference and Test TF with approximations (800 points, linear-linear view)

The advantage of this approach is that slight differences between TF are easily measurable because corresponding poles are recognisable in an objective way: Shifts of resonances can be read directly from a diagram like shown in Figure 12. It illustrates distribution of discrete resonance frequencies over the whole frequency range. The red curve reflects the reference TF and is therefore a straight line. The blue curve corresponds with the test TF. The green curve displays absolute frequency deviations of resonances.

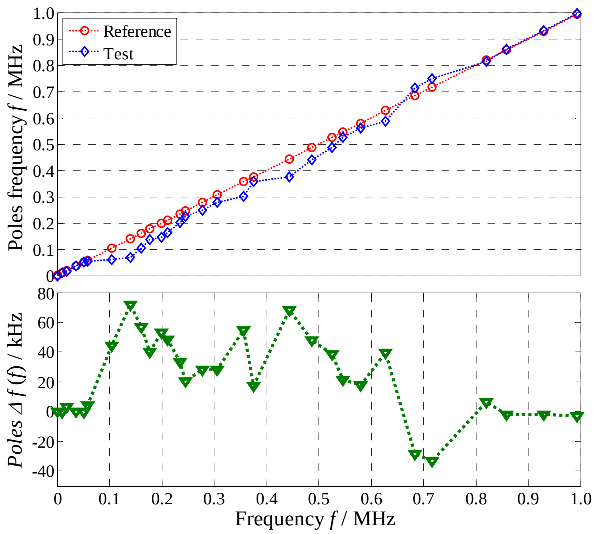

Figure 12: Comparison of frequency responses using pole frequencies of fitted rational function models.

Interpretation of slight deviations between frequency responses is the most challenging task of FRA. The found analytical representations are a first starting point for further algorithms contributing to automatic assessment of FRA measurements.

6. CONCLUSION

There is a need for algorithms helping to assess FRA measurements. The aim is to find an objective way to compare measured frequency responses of power transformers.
The advantage of a mathematical formulation of a TF over an array of hundreds of measured function values is the reduction of complexity.
Feasibility of approximation of measured frequency responses by rational functions is shown. The presented algorithm is based on the proven Vector Fitting method.
Mechanics of complex function approximation are predicated on relocation of poles of a predefined rational fitting function.
Loss of information for the calculated model of a TF is not acceptable. Accuracy of the fitting result has to be in the range of reproducibility imprecision of measurements. RMSE is a suitable measure for the quality of a fitting result and should be in the per mill range in order to fulfil accuracy requirements. Main resonances in measured TF should be captured with a frequency deviation of less than the recorded frequency resolution.
The procedure of TF approximation with Vector Fitting is improved by pre-estimation of the needed model complexity along with optimized starting pole distribution.
The developed algorithm was demonstrated using measured FRA data. Future interpretation algorithms may incorporate comparisons of pole patterns of fitted analytical models.

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