The Voltage and Time Parameter Measurement Uncertainties of a Large Damped Capacitor Divider due to its Non-ideal Step Response

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Abstract: Limited frequency response of impulse voltage dividers contributes to the measurement errors of the peak voltage and time parameters. This paper investigates the convolution method for the determination of the errors of lightning impulse parameters due to non-ideal step responses. The step response of a damped capacitor divider with a voltage rating of 2.2 MV was measured and analytical lightning impulse waveforms were used in the analysis. The paper concludes that the errors calculated with the convolution method can either be used for correcting the measured impulse parameters or as uncertainty components for estimating the measurement uncertainties of the impulse parameters.

1 INTRODUCTION

The step response parameters measured according to the current IEC Standard 60060-2 [1] serve mainly as finger prints of the dynamic performance of dividers in the context of measurement uncertainty estimation. The relationships between the measured step response parameters and their effects on the measured lightning impulse waveforms are not straightforward. However, if the unit step response function of the divider is known, then a lightning impulse waveform at the output of the divider can be calculated from the convolution of the derivative of input impulse waveform and the unit step response. To evaluate the errors introduced by the divider to lightning impulses of a particular waveform, the waveform can be constructed analytically and used as the input impulse. The waveform at the output of the divider can be calculated using the convolution method, and the errors introduced by the divider can then be calculated from the difference of the input and the output waveforms.

The de-convolution method, on the other hand, can be used to calculate the input waveform from the output waveform. It can also be used for checking the convolution algorithm developed.

This paper investigates the numerical procedures and technical issues in implementing the convolution and de-convolution methods. The measured step response of a damped capacitor divider as well as artificial step responses are used in the calculations. Analytical waveforms with varying front times and time-to-half values [2] are used. The errors in the impulse parameters due to the non-ideal step responses are calculated for a range of different step responses. The use of the calculated errors for correcting the measured impulse parameters and for estimating measurement uncertainties is also discussed.

2 THE CONVOLUTION METHOD

If the unit step response function of the divider is $g(t)$ and the input voltage is $V_{in}(t)$, then since $g(t)$ is causal and time-invariant, the output voltage function $V_{out}(t)$ can then be obtained by the following convolution integral assuming that $V_{in}(t)$ starts at $t = 0$ [3]:

$$V_{out}(t) = \int_0^t V_{in}^{'}(\tau) g(t-\tau) \cdot d\tau \qquad (1)$$

If $g(t)$ and $V_{in}(t)$ are sampled with the same interval and the sample number of $g(t)$ is less than that of $V_{in}(t)$, the continuous convolution integral (4) reduces to the causal form of the discrete convolution sum:

$$V_{out}(i) = \sum_{k=0}^{i} V_{in}^{'}(k) \cdot g(i-k) \cdot \Delta t \,, \quad i=0,\ldots,n-1 \qquad (2)$$

where n is the total number of samples of the input impulse waveform and Δt is the sampling interval.

Equations (1) and (2) indicate that the convolution can be performed for the derivative of the input impulse waveform and the unit step response. To ensure the accuracy of the derivative, a smooth input impulse waveform can be conveniently constructed.

In this paper, the convolution was performed with a commercial software package which implements the discrete convolution as described by equation (2).

3 THE STEP RESPONSES

Measured step responses as well as artificially created step responses are used in the calculation. The step response of a large damped capacitor divider with a lightning impulse voltage rating of 2.2 MV was measured first. The divider includes a high-voltage divider and secondary low-voltage divider, which is also a damped capacitor divider. The high-voltage divider was compensated [4] at its low-voltage arm for the temperature dependency of the capacitance of the paper/oil insulated high-voltage capacitors. A high output capacitance of 7 μF for the secondary divider was used to give a sufficiently high time constant when used with a digitiser input impedance of 1 MΩ for both lightning as well as switching impulses.

The height of the divider was approximately 7.9 metres. The step response was measured with the step voltage generator placed on the laboratory floor at a location where the conductor connecting the output terminal of the step generator and the input of the high-voltage divider was at an angle of 45° to the laboratory floor. The connecting conductor was an aluminium tube of 25 mm outside diameter. The low voltage ends of both the step generator and the divider were connected with a copper strip connected to the laboratory earth. The width of the copper strip was 200 mm. A photograph of the arrangement for the step response measurement is shown in Fig. 1.

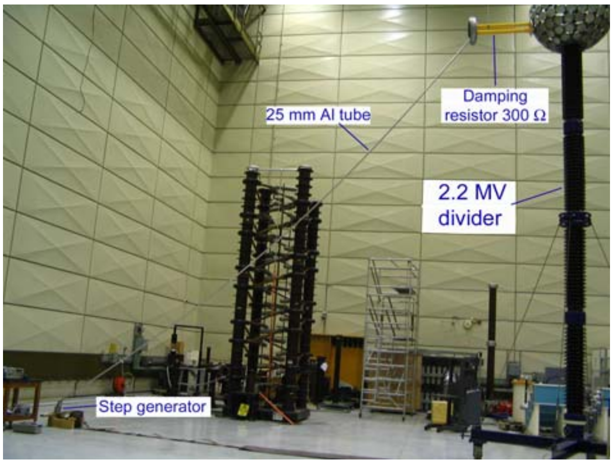

Fig. 1 Arrangement of the step response measurement

A recorded step response trace is shown in Fig. 2. The total length of the record is 100 μs and the length after the start of the step is 90 μs. The sampling interval used is 2 ns. The peak location of the first overshoot is approximately 150 ns after the start of the step response. The first negative peak of the oscillation is approximately 250 ns after the start of the response.

The calculated step response parameters are listed in Table 1. The calculation was performed with the origin set to the real start of the step instead of the point defined as the virtual origin according to IEC 60060-2 [4]. The overshoot amplitude was approximately 27%. The initial distortion was zero as a consequence of using the true start as the origin.

The epoch is the time interval within which the reference level is calculated. The time of the start of the measured step is the reference for the time limits of the epoch. The reference level is defined by Clause 3.7.3 of IEC 60060-2 [4] as the mean value of the step response taken over a Nominal Epoch.. The reference level is used for normalising a measured step response into a unit step response. Therefore the average response level of a unit step response within the epoch of the reference level, which will be referred to as the reference unit level in the following sections, is unity. Also in the following sections, the whole section of the normalised response record after the transition of the step will be referred to as the unit level. The unit level of a unit step response outside its reference epoch is not always unity, i.e., the unit level may not be constant.

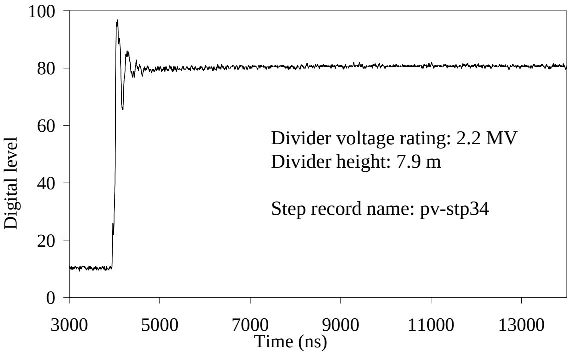

Fig.2 The initial part of the measured divider step response before normalised to unity.

Table 1 Calculated step response parameters

Epoch (μs)	T_N (ns)	T_α (ns)	t_s (ns)
20 to 22	1.0	55.9	2222
0.8 to 3.0	40.6	55.8	160.0

The results in Table 1 show that values of experimental response time T_N and the settling time t_s are strongly influenced by the epoch used for determining the reference unit level. This is because the unit level is not constant despite the effort of frequency compensation and use of high output capacitance. It can be concluded from these results that the output impulse computed from the convolution using the measured unit step response will also change with the selection of epoch for practical dividers.

To evaluate the deviations of the output impulses with slower step responses, artificial step responses with different values of experimental response time T_N [1] were also created and used for convolution with the input impulses. The unit level of artificial step responses was made constant. The artificial step is made with a smooth linearly rising front of a given slope which gives a certain T_N value. For instance, a step with a front rising from the zero level to the unit level in 100 ns gives a T_N value of 50 ns.

4 CONVOLUTION USING MEASURED UNIT STEP RESPONSES

Smooth analytical impulses were created and used as the input impulses for convolution with the unit step response. A range of different unit step responses were created from normalising the measured step response ‘pv-stp34’ as shown in Fig. 2 using different epochs.

To normalise the measured discrete step response array *s(n)* into a discrete unit step response array *g(n)* for convolution, the zero level of the measured step before the start of the step had to be determined. The initial part of the record before the start of the step is averaged to obtain the zero level $L_{0.}$ The reference level L_R is then determined by averaging the levels within the chosen epoch. The last sample point counted backwards with time above L_0 is chosen as the starting sample of the step response. The samples of *s(n)* from the starting point, *i=0*, until the end of the sample record, *i=m-1*, are then normalised by the difference between the reference level L_R and the zero level L_0, i.e.,

$$g(i) = \frac{s(i)}{L_R - L_0}, \quad i = 0,\ldots,m\text{-}1 \qquad (3)$$

This procedure effectively removes the samples before the start of the step from the unit step response. This removal is valid and necessary because the delay of the system response in a divider can be considered negligible. If the samples before the start are not removed, a delay equal to the time before the start of the step will be introduced to the convolved impulse output.

A comparison of an input impulse and its counterpart at the output calculated by convolution is shown Fig. 3. The slower start of the output impulse due to the non-ideal step response can be observed. Also the waving front of the output can also be seen. This waving deformation is due to the oscillation in the step response at the beginning of the unit level.

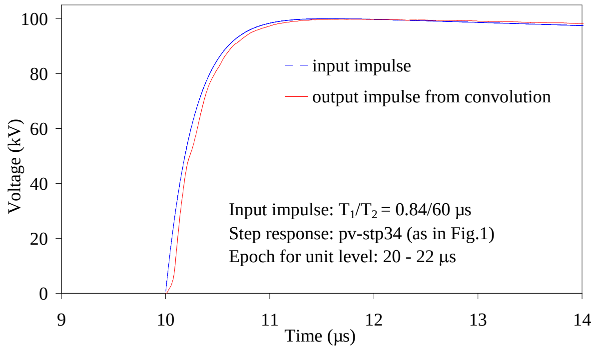

Fig. 3 Comparison of an input impulse and the output impulse obtained by convolution

Fig. 4 and Fig. 5 show the errors of T_1 values and the peak voltage values V_p of the output impulses respectively. The error in a parameter of the output impulse is calculated with the value of the corresponding input impulse as the reference. A positive error means the value of the output is higher than the value of the input. Three unit step responses obtained using three different epochs were used in the calculations. These results show that the error in T_1 changes with the T_1 value of the input impulse, i.e., with the gradient of the impulse front. This can be explained by two effects. First, a non-ideal step response would generally slow down an impulse with faster front more. The second effect comes from the oscillation, which deforms the front of the output impulse into a waving one (Fig. 3). The relative time location of the oscillation peak of the step response to the time locations of $0.3V_p$ and $0.9V_p$ voltage points of the impulse for T_1 calculations [2] determines the change in T_1. This means that, if significant oscillation is present in the step response, the step response would not necessarily cause a smaller T_1 error in a slower impulse than in a faster impulse. It also can be seen from Fig. 4 that the selection of epoch does not change the deformation of the impulse front. This is because the differences in the reference unit levels calculated from different epochs are small. Therefore the front parts of the unit steps, including the oscillations, are almost the same in shape.

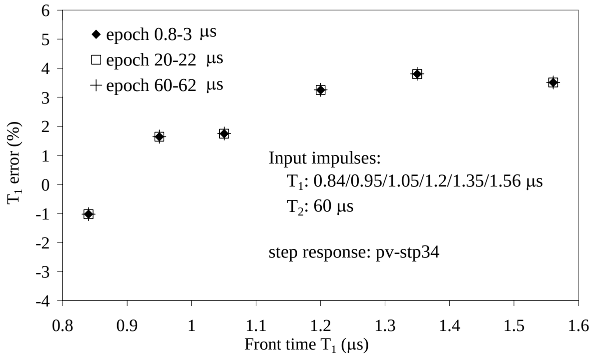

Fig. 4 Errors of output T_1 for different input waveforms

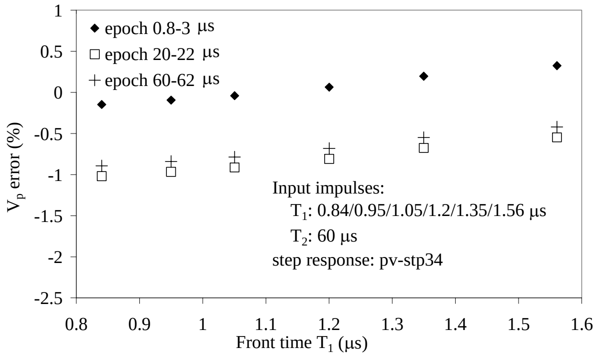

Fig.5 Errors of output V_p for different input waveforms

The error of the peak voltage V_p of the output impulse, however, changes significantly with the selection of the epoch (Fig. 5). The change in the V_p error due to change of T_1 with given epoch can also be seen from Fig.5. A minimum error of -0.04 %, with an epoch between 0.8 and

3.0 μs, is obtained for the impulse having an input T_1 value of 1.05 μs. This is of course not a universal outcome, rather a result due to the fact that time location of the impulse peak happens to match the selected epoch. The output V_p values calculated with the epoch of 20 to 22 μs and the epoch of 60 to 62 μs are lower than the value with the epoch of 0.8 to 3.0 μs by 0.87% and 0.74% respectively. These differences of V_p values match exactly with the differences between the reference unit levels of the three epochs. If the average unit level within the epoch of 0.8 to 3.0 μs is taken as the reference unit level, then the average unit levels within the epochs of 20 to 22 μs and 60 to 62 μs would be 1.0087 and 1.0074 respectively.

5 CONVOLUTION WITH ARTIFICIAL STEP RESPONSES

The results in Section 4 indicate that, even for a large damped capacitor divider, the accuracy requirements [1] for impulse testing can still be easily met even if the measured step response parameters are outside of the limits recommended in the current IEC 60060-2[1]. To further investigate the errors of the impulse parameters with slower step responses, artificial step responses with larger values of experimental response time T_N were created and tested. Details of the artificial step response are described in Section 3.

Fig. 6 and Fig. 7 show the errors of the T_1 and V_p of the output impulses respectively with increasing value of T_N of the step response.

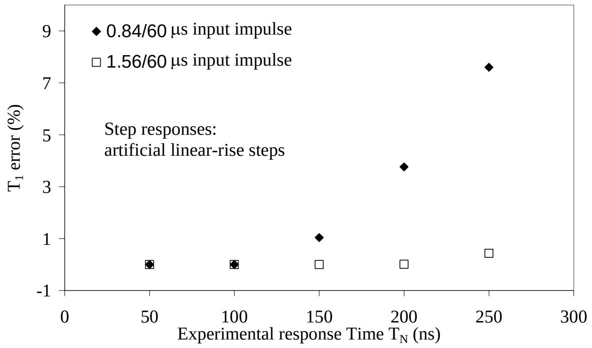

Fig. 6 Errors of T_1 values when T_N values are high

With the T_N values up to 250 ns, errors of T_1 are not significant for the input impulse with a T_1 value of 1.56 μs, which is the upper limit for a standard lightning impulse [2]. For an input impulse with a T_1 value of 0.84 μs (the lower limit for the standard lightning impulse), T_1 errors are between +1 % and +8 % for T_N values ranging from 150 to 250 ns. When T_N is 100 ns or lower, T_1 error is negligible for the 0.84/60 impulse. For T_N values up to 250 ns, the errors of the peak voltage are less than 0.05%, which is negligible for the purpose of most impulse voltage tests.

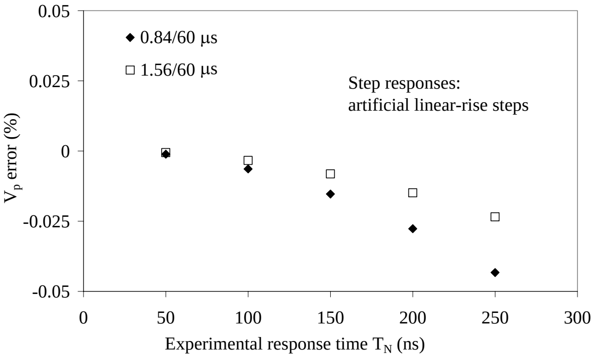

Fig. 7 Errors of V_p values when T_N values are high.

6 IMPULSES WITH OVERSHOOT

Under some test conditions, an overshoot of the lightning impulse cannot be avoided. The overshoot normally contains components with frequencies higher than those in the main impulse. It is necessary for a high-voltage divider to reproduce the overshoot without distortion, even though the overshoot is modified according to its frequency contents after the impulse waveform is recorded [2].

An analytical input impulse waveform with superimposed overshoot is generated from the calculation of the output of an impulse generation circuit. The relative overshoot amplitude is approximately 9% and the overshoot frequency is known from the generation circuit to be 650 kHz. The analytical impulse was first convolved with the measured unit step response normalised with a normalisation epoch of 0.8 to 3.0 μs. The output impulse calculated from the convolution, together with its input counterpart, is shown in Fig. 8. The T_1 value and the peak voltage value V_p of the output impulse decreased by 1.1 % and 0.37 %, respectively. These results are similar to those obtained by the overshoot-free impulse shown in Fig. 4 and Fig. 5. The error in V_p is larger than that of the 0.84/60 overshoot free impulse (Fig. 5). This is because the location of the peak is earlier and now misaligned with the normalisation epoch of 0.8 to 3.0 μs.

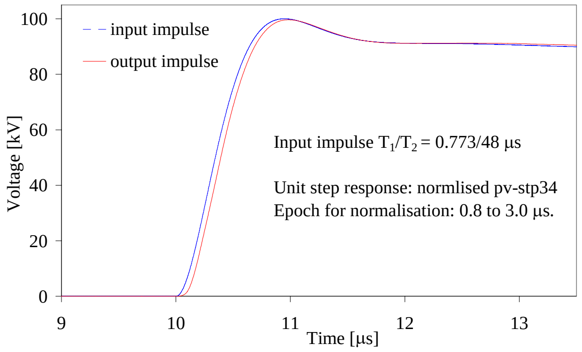

Fig. 8: Comparison of the input and output impulses with overshoot

The artificial step responses described in Section 3 are also used for evaluation of the impulse with superimposed overshoot. Fig. 9 and Fig. 10 show the T_1 errors and the V_p errors of the output impulses respectively. The effect on T_1 and V_p is now more significant because of the presence of components of higher frequencies in the overshoot of the impulse waveform.

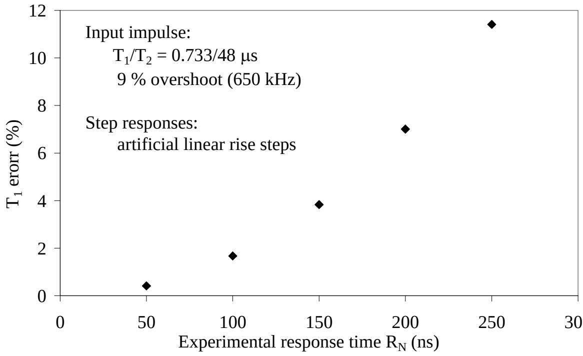

Fig. 9 Errors of T_1 values due to artificial step responses

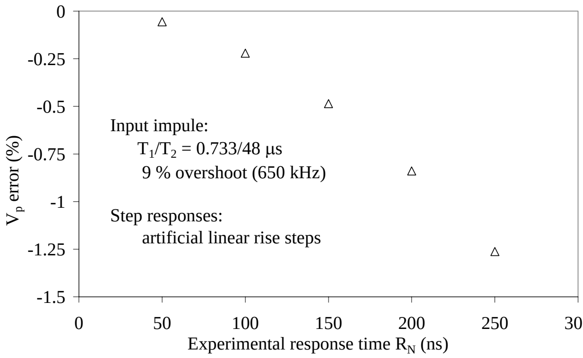

Fig. 10 Errors of V_p values due to artificial step responses

7 DE-CONVOLUTION

A de-convolution procedure was also developed to obtain the input impulse from the convolved output impulse.

With this procedure, the derivative of the discrete input impulse array $V_{in}^{'}(n)$ is first calculated by

$$V_{in}^{'}(n) = InvFFT\left(\frac{V_{out}(f)}{G(f)}\right) \qquad (3)$$

where $V_{in}^{'}(n)$ is the de-convolved derivative of the input voltage array, $V_{out}(f)$ the Fourier Transform of the output voltage array obtained by convolution, $G(f)$ the Fourier Transform of the unit step response $g(n)$ and $InvFFT$ the Inverse Fast Fourier Transform.

The voltage array of the input impulse, $V_{in}(n)$, is then obtained by a discrete integration algorithm using the Simpson's Rule, i.e.,

$$V_{in}(i) = \sum_{j=0}^{i}\left[V_{in}^{'}(j-1) + 4V_{in}^{'}(j) + V_{in}^{'}(j+1)\right]\times\frac{dt}{6} \qquad (4)$$

The measured step response, pv-stp34, was again used for the calculation. The de-convolution calculation performed here is mainly for validation of the convolution algorithm used in the previous sections. Table 2 shows the parameters of an original input impulse, parameters of the convolution output and the parameters of the de-convolution version of the impulse calculated from the original by convolution. It can be seen that the original is re-constructed accurately by the full convolution and de-convolution cycle. The results confirm that the convolution algorithm used is correct.

Table 2 Comparison of an original, convolution and de-convolution impulse

	Original	Convolution	De-convolution
Vp	100.000	99.852	100.000
T_1 (µs)	0.84015	0.83159	0.84015
T_2(µs)	60.0169	60.91033	60.0159

Fig. 11 shows the comparison of an original input impulse with its output counterpart obtained by convolution with an artificial step response. Fig. 12 shows the comparison of the convolved output with its de-convolved counterpart. It can be seen that the de-convolved impulse becomes the same impulse as the original input impulse.

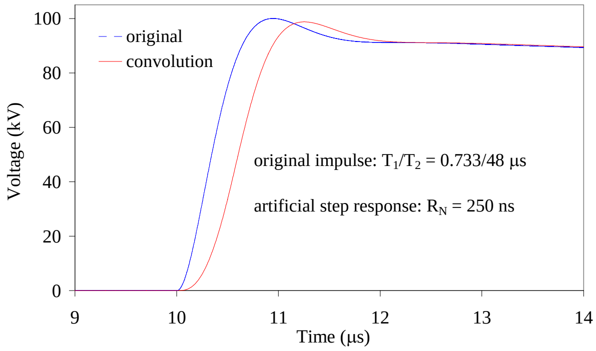

Fig. 11 An original input impulse and its convolved output

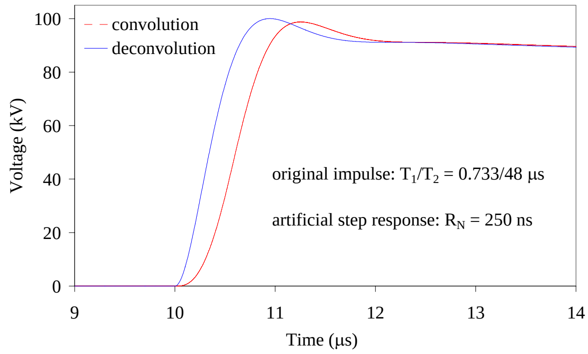

Fig. 12 The convolved output impulse and the replica of the input obtained by de-convolution of the output

8 CORRECTION AND UNCERTAINTY OF IMPULSE PARAMETERS

The errors of the impulse parameters obtained by the convolution may either be used for estimating the measurement uncertainties of the impulse parameters or, more ideally, for correcting the measured parameters. The use of calculated errors for parameter correction depends on a range of measurement conditions.

If no overshoot/oscillation is present in the step response, the T_1 error will always be positive, which means the T_1 of the output is greater than T_1 of the input. The error will increase as T_N increases. With a given T_N value, the error will increase as input T_1 decreases. The T_1 error is almost totally unrelated to the selection of epoch even if the unit level is not constant. Therefore, when overshoot is not present in the measured step response, the calculated T_1 error may be used for correcting T_1 of a measured output. However, significant T_1 errors only occur for T_N well above 100 ns (Fig. 6), which is significantly higher than the T_N values of most practical dividers. Therefore, in reality, no significant T_1 correction or uncertainty contribution is applicable if the measured step response is free of oscillation/overshoot.

In cases such as 'pv-stp34' (Fig. 2) where significant oscillation/overshoot is present, correcting T_1 according to the calculated T_1 error is only possible if waveform of input impulse can be closely matched to that of the impulse being measured. Waveform matching is only possible for some cases such as measurement of the output of an impulse calibrator using an impulse voltage probe, where the output waveform of the calibrator is normally well defined. Therefore the calculated T_1 errors, in most cases, may only be used for estimating the uncertainty [5] of the measured T_1.

As for the peak voltage V_p, with T_N in the normal range of practical dividers, the error is insignificant if the unit level is constant (or near constant) or the epoch for the reference unit level is the same as the epoch for the impulse peak voltage measurement [1]. However, the unit level of a practical divider, especially the damped capacitor type, is often not constant. Also, the epoch for the peak voltage measurement may not always match the epoch of the reference unit level of the unit step response. For example, if the scale factor of the divider is measured with DC voltage, the scale factor for peak voltage measurement is then in fact defined for an epoch at time infinity. The V_p error calculated by convolution with the unit step response normalised to a certain epoch, say 0.8 to 3.0 μs, will then not be the error of the measured peak voltage using the DC scale factor if the unit level is not constant.

If the divider scale factor is measured by the step method [6] and the epoch for calculating the scale factor is the same as the epoch for calculating the unit step response, then the error calculated from the unit step response would be the true measure of the error in the peak voltage measured by the divider. In summary, the epoch for calculating the unit step response has to match the epoch for obtaining the scale factor to ensure that the calculated V_p error can be used for correcting the measured V_p or in estimating the uncertainty of measured V_p.

9 CONCLUSION

The output of an impulse divider can be accurately calculated by the convolution method using the measured unit step response of the divider. The convolution algorithm used has been verified to be accurate using a de-convolution algorithm. The front time of the output impulse is strongly influenced by the oscillation present on the front of the step response. The peak voltage of the calculated output impulse changes significantly with the selection of the epoch used for normalisation of the unit step response if the unit level is not constant. Therefore the epoch of the unit step response has to match the epoch of the scale factor measurement before the peak voltage error calculated by the convolution method can be used for the purpose of correction or uncertainty estimation of the measured peak voltage.

10 REFERENCES

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