

## Improved Top-oil Temperature Model for Unsteady-State Conditions of Power Transformers

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**Abstract:** This paper presents a long-term period investigation of some dynamic top-oil temperature models to be used in an on-line monitoring and diagnostic system for power transformers. Some parameters in the models were estimated by the least square optimization technique. The calculation of top-oil temperature was performed at varying load current for a 150 MVA-ONAF and 333 MVA-OFAF Transformer units. The effect of varying conditions of the cooling unit and the unsteady-state behaviour of load current and ambient temperature on the transformer thermal behaviour has been investigated.

**Key Words-** Power transformer, thermal model, top-oil temperature, on-line monitoring

### NOMENCLATURE

$\theta_{TO}$	top-oil temperature, °C
$\theta_{TO,U}$	ultimate top-oil temperature for load L, °C
$\theta_{TO,I}$	initial top-oil temperature for t = 0, °C
$\theta_{amb}$	ambient temperature, °C
$\Delta\theta_{TO}$	top-oil rise over ambient temperature, °C
$\Delta\theta_{TO,I}$	initial top-oil rise for t = 0, °C
$\Delta\theta_{TO,R}$	top-oil rise at rated load, °C
$\Delta\theta_{TO,U}$	ultimate top-oil rise for load L, °C
$\Delta t$	sampling period, hours
$C_{th}$	thermal capacitance of the transformer, W-hours/K
K	ratio of load L to rated load
$P_S$	short circuit loss, W
$P_N$	no-load loss, W
$P_T$	total loss, W
R	ratio of load loss to no-load loss at rated load
$R_{th}$	thermal resistance of the transformer, K/W
$c_p$	specific heat capacity, W-hours/kg·K
m	weight, kg
n	oil exponent
$\tau_{TO}$	top-oil time constant, hours

### INTRODUCTION

At present, for economic reasons, there is an increasing emphasis on keeping transformers in service longer than in the past. The basic criterion, which limits the transformer load ability and usable life, is partially determined by the ability of the transformer to dissipate the internally generated heat to its surrounding. Therefore, the knowledge of the transformer thermal performance could lead to an improvement of the utilization of transformers. By on-line comparison of a measured quantity such as top-oil temperature and a calculated value, which is obtained by means of the physical model, some rapidly developing failures such as the malfunction of pumps or fans can be detected [1].

### TOP-OIL TEMPERATURE MODELS

As a thermal model for an on-line monitoring system, the IEEE/ANSI C57.115 standard top-oil rise temperature model [2] may be chosen as a fundamental model for the prediction of top-oil temperature (**Model A**). This model is based on the concept that the change of top-oil temperature rise over ambient temperature is caused by change in loading condition, which is governed by the following first-order differential equation.

$$\tau_{TO} \frac{d\Delta\theta_{TO}}{dt} = -\Delta\theta_{TO} + \Delta\theta_{TO,U} \quad (1)$$

Which has the solution as follows:

$$\Delta\theta_{TO} = (\Delta\theta_{TO,U} - \Delta\theta_{TO,I}) (1 - \exp^{-\Delta t/\tau_{TO}}) + \Delta\theta_{TO,I} \quad (2)$$

where 
$$\Delta\theta_{TO,U} = \Delta\theta_{TO,R} \left[ \frac{K^2 R + 1}{R + 1} \right]^n \quad (3)$$

and 
$$\tau_{TO} = \Delta\theta_{TO,R} \frac{C_{th}}{P_T} \quad (4)$$

The top-oil temperature is then given by

$$\theta_{TO} = \Delta\theta_{TO} + \theta_{amb} \quad (5)$$

However, it can be noticed that this fundamental model has the limitation that it does not accurately account for the effect of daily variations in ambient temperature, and is therefore not applicable for an on-line monitoring system. The MIT group [3] has later proposed the

modified top-oil temperature model (**Model B**) based on a concept originally developed from the IEEE top-oil rise temperature model by considering the ambient temperature at the first-order characterization.

$$\tau_{TO} \frac{d\theta_{TO}}{dt} = -\theta_{TO} + \theta_{amb} + \Delta\theta_{TO,U} \quad (6)$$

Using forward Euler approximation for the time derivative:

$$\frac{d\theta_{TO}[k]}{dt} \approx \frac{\theta_{TO}[k] - \theta_{TO}[k-1]}{\Delta t} \quad (7)$$

And by applying linear regression technique for the force cooling state  $n = 1$ , the equation is expressed as below:

$$\begin{aligned} \theta_{TO}[k] = & \frac{\tau_{TO}}{\tau_{TO} + \Delta t} \theta_{TO}[k-1] + \frac{\Delta t}{\tau_{TO} + \Delta t} \theta_{amb}[k] \\ & + \frac{\Delta t \Delta \theta_{TO,R}}{(\tau_{TO} + \Delta t)(R+1)} (K[k])^2 + \frac{\Delta t \Delta \theta_{TO,R}}{(\tau_{TO} + \Delta t)(R+1)} \end{aligned} \quad (8)$$

with the simplified model

$$\theta_{TO} = K_1 \theta_{TO}[k-1] + (1-K_1) \theta_{amb}[k] + K_2 K[k]^2 + K_3 \quad (9)$$

The parameter  $K_1$ ,  $K_2$  and  $K_3$  are obtained from the parameter estimation method. However, this simplified model is under the assumption of the cooling type  $n = 1$ .

Therefore, under the consideration of two important thermal parameters of a cooling system of transformer, its heat capacity and its thermal conductivity were considered in the modified model (**Model C**). Base on an assumption that all losses are transferred to an environment via a thermal resistance ( $R_{th}$ ) of the cooling equipment and by taking the variation of ambient temperature into consideration, the modified model was proposed as below [1,4].

$$\theta_{TO} = (\theta_{TO,U} - \theta_{TO,I}) (1 - \exp^{-\Delta t/\tau_{TO}}) + \theta_{TO,I} \quad (10)$$

$$\text{where } \theta_{TO,U} = \theta_{amb} + (P_S K^2 + P_N) R_{th} \quad (11)$$

$$\text{and } \tau_{TO} = C_{th} R_{th} \quad (12)$$

## PARAMETER ESTIMATION

Three models as mentioned above (Equ. 2, 9, 10) were examined in this paper using the data from two different types of three phase transformer units. The specifications of these two transformers are shown in Table 1. The required measured data were obtained from an on-line monitoring system MS 2000 supplied by the AREVA Energietechnik GmbH. The data interval is 15 minutes. Furthermore, it has been noted that the status of the cooling units of Transformer 1 (**Tr1**) remained constant during the investigated time period (number of operating fans = 2). Whereas, the cooling state of Transformer 2 (**Tr2**) is varied between two states (number of operating fans = 4, pumps = 2 and number of operating fans = 8, pumps = 4).

Table 1: Power transformer main characteristics

	Tr1	Tr2
Rated power	150 MVA	333 MVA
Rated voltage	220 kV	400 kV
Short-circuit loss	414.2 kW	400 kW
No-load loss	67.52 kW	47 kW
Type of cooling	ONAF	OFAF

Based on the specific heat capacity  $c_p$ , density  $\rho$ , and the volume  $v$  of the observed parts of the transformer, the approximated range of thermal capacitance  $C$  can be expressed by the following equation:

$$C = c_p \rho v \quad (13)$$

Table 2 shows the specific thermal capacitance of the observed components of a transformer, which are included oil, winding, core, tank and clamps. Thus, the thermal capacity in this paper can be calculated from:

$$C_{th} = m_{oil} c_{p,oil} + m_{Cu} c_{p,Cu} + m_{Fe} c_{p,Fe} + m_{Steel} c_{p,Steel} \quad (14)$$

Table 2: Specific thermal capacitance

Material	Cp [W-hours/kg·K]
Oil	0.5360
Copper [Cu]	0.1066
Eisen [Fe]	0.1288
Steel	0.1332

For Tr1:  $C_{th1} = 42509$  W-hours/kg·K

For Tr2:  $C_{th2} = 59668$  W-hours/kg·K

Due to an uncertainty associated with some manufacturer supplied parameters and the complicated transformer configuration, some model parameters have been optimized. The least square technique was applied in the models, in order to estimate the parameters from each model as classified in Table3. The least square technique equation is expressed as below.

$$\min_x \frac{1}{2} |F(x, xdata) - ydata|_2^2 = \frac{1}{2} \sum_{i=1}^m (F(x, xdata_i) - ydata_i)^2 \quad (15)$$

In this equation  $xdata$  is the given input data,  $ydata$  is the observed output, and the coefficients  $x$  is found from the best fit to the equation.

Table 3: Parameter classification

	Model A	Model B	Model C
Input constants	1.Short-circuit loss 2.No-load loss 3.Thermal cap.	- - -	1.Short-circuit loss 2.No-load loss 3.Thermal cap.
Input variables	1.Load factor 2.Ambient temp.	1.Load factor 2.Ambient temp.	1.Load factor 2.Ambient temp.
Estimated parameters	1.Top-oil rise temp. at rated load	1.K1 2.K2 3.K3	1.Thermal resistance

For the purpose of comparing the results from different periods of time within a year, all data were separated into dataset. Each dataset covers two weeks. From each of dataset, the parameter  $\Delta\theta_{TO,R}$ ,  $K_1$ ,  $K_2$ ,  $K_3$  and  $R_{th}$  from the three models as mentioned above were estimated.

The top-oil temperature was selected as a model output. By applying the estimated parameters in the models, the difference between the measured top-oil temperature and the calculated top-oil temperature was calculated as an error of the model.

Table 4 presents some results of the estimated parameters from Tr1 for different learning periods of two weeks. Additionally the parameters were also estimated by using a whole year data. These estimated parameters are given as total year parameters.

Table 4: Estimated parameters from Tr1 during different periods of time in a year

Time	Model A	Model B			Model C
	$\Delta\theta_{TO,R}[K]$	K1	K2	K3	$R_{th}[k/W]$
01.01.03-15.01.03	63.3	0.964	1.270	0.730	$1.369 \cdot 10^{-4}$
01.03.03-15.03.03	62.7	0.974	1.136	0.381	$1.357 \cdot 10^{-4}$
01.05.03-15.05.03	60.4	0.963	1.279	0.716	$1.298 \cdot 10^{-4}$
01.07.03-15.07.03	58.4	0.962	1.330	0.688	$1.254 \cdot 10^{-4}$
01.09.03-15.09.03	62.5	0.963	1.272	0.729	$1.351 \cdot 10^{-4}$
01.11.03-15.11.03	63.9	0.964	1.270	0.729	$1.384 \cdot 10^{-4}$
<b>Total Year</b>	<b>61.7</b>	<b>0.963</b>	<b>1.272</b>	<b>0.729</b>	<b><math>1.330 \cdot 10^{-4}</math></b>

Results show that the estimated parameters depend on the investigated time period. For example, the model A parameter estimation during 01.01.03-15.01.03 gives a top-oil rise at rated load  $\Delta\theta_{TO,R}$  of 63.3°C and during 01.05.03-15.05.03 a value of 60.4°C. To check the accuracy of the examined models and their estimated parameters, the top-oil temperature was calculated by means of the respective model and compared with the measured top-oil temperature in the learning period. The average error between these values is presented as two – week value in Fig.1, Fig.2 and Fig.3 for model A, B and C respectively. The highest error in all models occurs during the January period and is app. 3K.

### Results for Transformer 1

In order to achieve a simple adaptation of the thermal model for the use in an on-line monitoring system, the parameters must not change during the course of a year. Therefore, the error is investigated by using the total year parameters for the top-oil temperature calculation within all two weeks intervals. It is presented and compared with the error from two-week value also in Fig.1, Fig.2 and Fig.3 for model A, B and C respectively.

It is found in model A and C that, in some periods of time e.g. 01.07.03-15.07.03, the temperature error using the total year parameters is much higher than the temperature error using the two-week parameters. Whereas the result from model B shows that the constant estimated parameters could be determined directly from the total year parameters, because the error is not increased substantially.

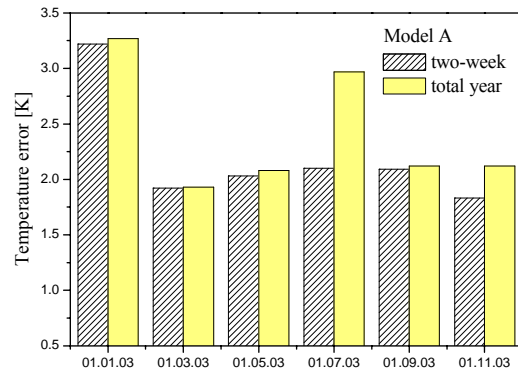


Fig.1 Error in the calculated top-oil temperature using two-week estimated parameters and total year estimated parameters applied in model A

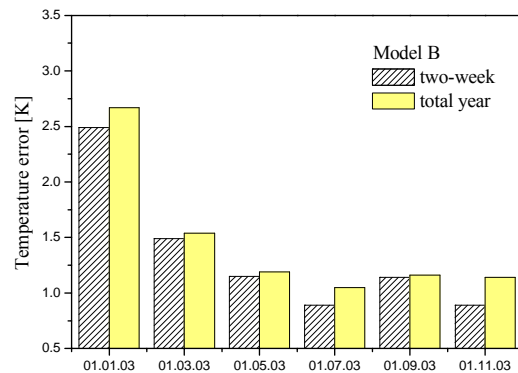


Fig.2 Error in the calculated top-oil temperature using two-week estimated parameters and total year estimated parameters applied in model B

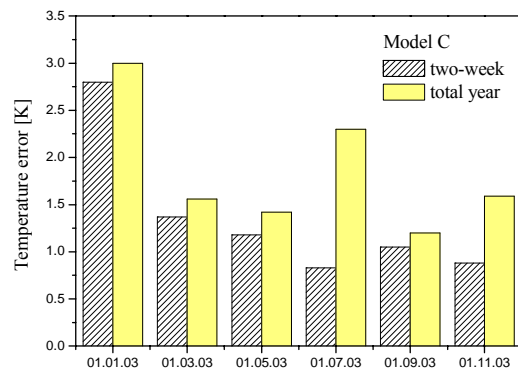


Fig.3 Error in the calculated top-oil temperature using two-week estimated parameters and total year estimated parameters applied in model C

Furthermore, the performance of each model is compared. The calculated errors using the total year estimated parameters from each model are illustrated again in Fig.4. It can be clearly seen that the difference between the calculated and measured values of top-oil temperature from model B is found to be the lowest. Whereas the temperature error calculated from model A is found to be the highest.

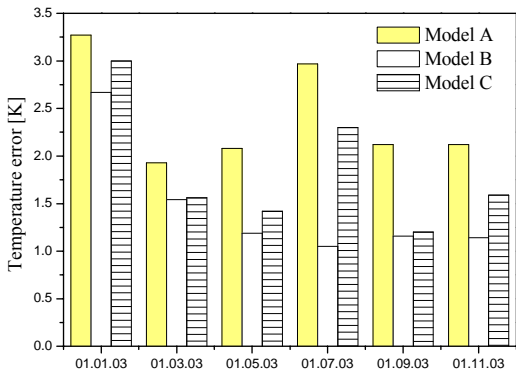


Fig.4 Temperature error comparison between different models

A plot of calculated and measured top-oil temperature during the time 01.08.03-15.08.03, which has the lowest temperature error in a year, is presented in Fig.5. It indicates that the top-oil temperature calculating from model B and from model C perform a very good result in capture the course of measured top-oil temperature along the period. Further, due to the assumption that they are known as the main factors that effect top-oil temperature, measured ambient temperature and load factor during this period shown in Fig.6 are considered. It can be noticed that, the loading condition is almost constant and ambient temperature varies in the normal course.

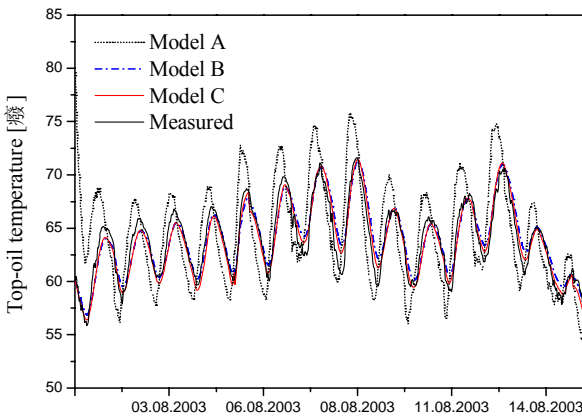


Fig.5 Top-oil temperature of Tr1 during the lowest error period

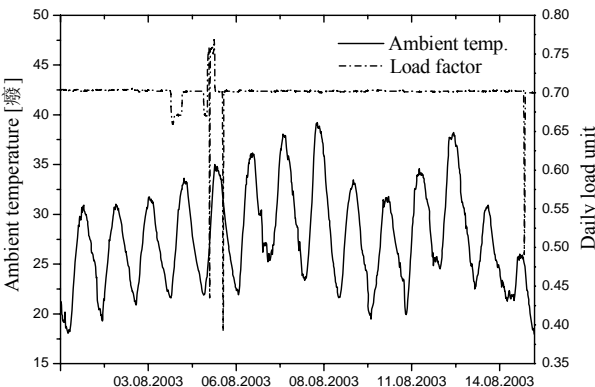


Fig.6 Load factor and ambient temperature of Tr1 during the lowest error period

When the highest error period is considered, Fig.7 and Fig.8 indicate that the transient behaviour of measured top-oil temperature is strongly influenced by the transient-state of ambient temperature. The maximum error of app. 5 K can be seen between 06.01.03 and 12.01.03 at low ambient temperatures. Basically all models show similar deviations from the measured top-oil temperature.

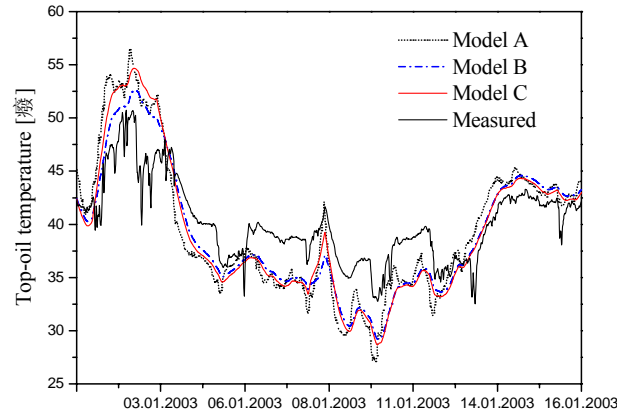


Fig.7 Top-oil temperature of Tr1 during the highest error period

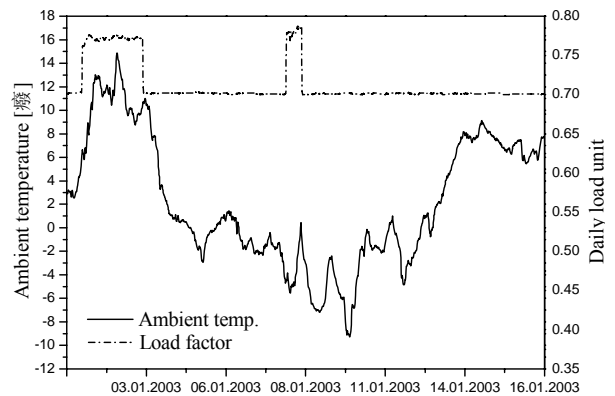


Fig.8 Load factor and ambient temperature of Tr1 during the highest error period

The relation between transient-state of ambient temperature and error from top-oil temperature calculation can be clearly seen from Fig.9. The dataset is obtained from 01.06.03-15.06.03 and is examined with model B. In this period, the ambient temperature appeared either in the steady-state or in transient-state. It can be noticed from the figure that during the state of instantly falling of the ambient temperature, the errors increase up to 9 K. Whereas the average temperature error in this period is at 1.4 K.

Basically the investigation of the three models shows a good agreement between calculated and measured data. Only in case of transient states of ambient temperature the error of top-oil temperature calculation appears to be significant. Furthermore, using model B and C achieve best results.

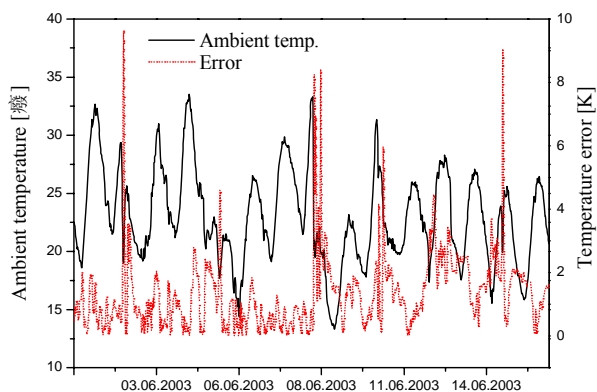


Fig.9 Influence of transient-state of ambient temperature on temperature error (Tr1, model B)

### Results for Transformer 2

According to the well-performed results of model B and C as shown before, these two models were examined in the next attempt. The unsteady-state of load current and cooling states of Tr2 were investigated. The operation states of pumps and fans are expressed in two sets. The first state is when the number of operating fans equals 4 and number of operating pumps equals 2. The second state is when the number of operating fans is 8 and the number of operating pumps is 4. During the investigated periods, the transformer always operates in these two operating states of fans and pumps. The parameter estimation method was applied for each operating state of pumps and fans separately, because the cooling performance is of course dependent on the operating state. The results of the parameter estimation for model B and model C during different periods of time are shown in Table 5 and Table 6 respectively.

Table 5: Estimated parameters for Tr2 from model B

Time	Fan =4, Pump =2			Fan =8, Pump =4		
	K1	K2	K3	K1	K2	K3
01.06.01-15.06.01	0.939	2.215	0.723	0.935	1.570	0.377
16.06.01-30.06.01	0.954	1.975	0.554	0.911	1.747	0.721
01.07.01-15.07.01	0.950	1.525	0.657	0.902	1.589	0.907
16.07.01-31.07.01	0.956	3.089	0.553	0.920	1.951	0.609

Table 6: Estimated parameters for Tr2 from Model C

Time	$R_{th}[k/W]$	
	Fan =4, Pump =2	Fan =8, Pump =4
01.06.01-15.06.01	$1.539 \cdot 10^{-4}$	$7.628 \cdot 10^{-5}$
16.06.01-30.06.01	$1.728 \cdot 10^{-4}$	$7.830 \cdot 10^{-5}$
01.07.01-15.07.01	$1.674 \cdot 10^{-4}$	$8.280 \cdot 10^{-5}$
16.07.01-31.07.01	$1.703 \cdot 10^{-4}$	$8.599 \cdot 10^{-5}$

Table 7: Error of top-oil temperature calculation from Tr2

Time	Temperature error [K]	
	Model B	Model C
01.06.01-15.06.01	1.2	1.1
16.06.01-30.06.01	1.1	1.6
01.07.01-15.07.01	1.0	1.6
16.07.01-31.07.01	0.9	1.2

Table 7 presents the error of top-oil temperature calculation for model B and C using the estimated parameters obtained from different operation states of pumps and fans. It can be seen that the both thermal models yield very good results basically. The results plotted from the period 16.07.01-31.07.01 by the referred models in Fig.10 agree with the measured values with a very good accuracy especially in the case of model B. This implied that these two thermal models are applicable for the top-oil temperature calculation both for transformer with the cooling type ONAF and OFAF. The plot of load factor and ambient temperature during the different operating states of pumps and fans can be seen from Fig.11

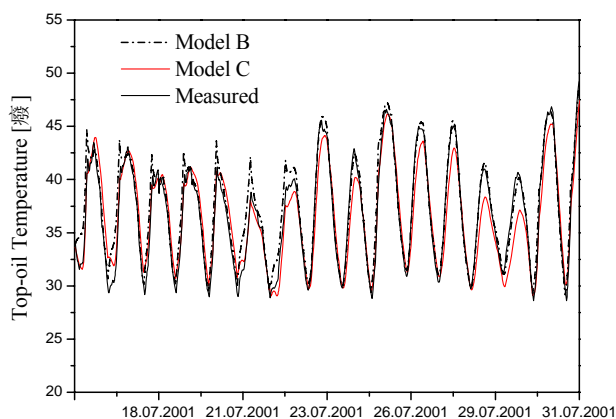


Fig.10 Top-oil temperature of Tr2 during the lowest error period

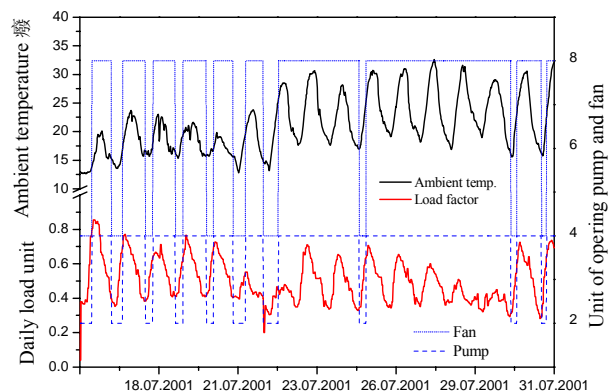


Fig.11 Load factor, ambient temperature and operating states of pumps and fans during the lowest error period of Tr2

### CONCLUSIONS

Three different top-oil temperature models were applied in two transformer units using the data from an on-line monitoring system during unsteady-state of load current and ambient temperature. Some constant parameters of the models were estimated from the least square optimization technique. Results indicate that the estimated parameters are not constant values. They vary by different periods of time of the year and vary by different operating states of pumps and fans. The applicability of the models was checked by calculation

of the error between calculated and measured top-oil temperature. Thermal model B and C have yielded the better result in top-oil temperature calculation than model A in long-term investigation. The average error in long-term supervision is below 2 K, which is sufficient for use in an on-line monitoring system. The results have revealed that the transient-state of ambient temperature has a negative effect on top-oil temperature calculation, whereas a transient-state of loading condition has a minor effect on the accuracy of the investigated models.

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